Physics-based modelling and animation of saccadic eye movement

Chris Papapavlou
University of Patras, Electrical and
Computer Engineering Department
University Campus, Rion
Greece 26504, Rion, Patras
chrispapapaulou@ece.upatras.gr

Konstantinos Moustakas
University of Patras, Electrical and
Computer Engineering Department
University Campus, Rion
Greece 26504, Rion, Patras
moustakas@ece.upatras.gr

ABSTRACT

In this paper we present a new approach in producing realistic saccade eye movement animations by incorporating anatomical details of the oculomotor system into the dynamics of the eye model. Unlike abstract models of the eye motor behaviour, we make use of a biomedical framework to effectively model the eye globe along with the three extraocular muscle pairs in efficient detail, that the application of the corresponding muscle activation signals, naturally results in realistic motions. That way, we avoid the need of explicitly providing trajectory information, and therefore simplify the process of eye animation. Regarding the calculations of the muscle activation signals needed to drive the animation in a way that imitates a real human eye, we are based on existing knowledge about the way that the nervous system utilizes the extraocular muscles during saccades.

Keywords
Eye animation, saccades, physics-based simulation.

1 INTRODUCTION

Among the human sensory organs, eyes stand out as they serve not only as an input organ, but play a non negligible role in human communication, since eyes are the spot that we are accustomed to look when talking to somebody. Hence, we are extremely sensitive at detecting even infinitesimal eye movements. That is why a non-realistic eye animation immediately impairs acceptance of computer animation.

1.1 Eye movements
The movements of the human eye are divided into several categories.

Fixation: This is not a movement, but the state when the gaze has been stabilized at the desired target and the eyes appear to be still. However, the eyes are never in a state of complete rest and even during fixation times they perform random jitter movements in order to satisfy the demand of the photoreceptors for non-constant stimulus.

Saccades: Quick, simultaneous movements of both eyes in the same direction which serve to bring the visual target in the centre of the retina, in the area called fovea where the detail and sharpness of the vision is maximized. This is necessary for actions like reading or driving where maximum visual resolution is needed.

Vestibulo-ocular reflex (VOR): Also known as oculocephalic reflex. A reflex eye movement that stabilizes images on the retina during head movement by producing an eye movement in the direction opposite to head movement, thus preserving the image on the center of the visual field.

Smooth pursuit: It is a common action of the eyes which allows us to precisely follow a moving visual target. Not to be confused with VOR movements which serve specifically to balance the head movement.

Replication of the aforementioned movements is a research area in computer graphics that receives significant attention during the latest years. However most approaches focus on the problem of fixation and smooth pursuit and neglect the significant saccadic eye motions. A review of the recent state-of-the-art follows in the next paragraph.

1.2 Related work
Previous work on eye modelling descends mainly from two different scientific communities. The first is the computer graphics community [6] [7], where the main focus is the production of realistic human-like eye movements, while the other is the biomedical community where the main interest is the study of

the eyes as a human organ aiming to gain a better understanding of the way eyes *work* with ultimate goal to find new solutions to eye diseases [14].

From a computer graphics point of view, notable works are [8] where an eye movement model has been developed and [2] where they achieved the automated production of eye movements using texture synthesis. Moreover, several approaches have been proposed in the past to model eye gaze of conversational avatars focusing mainly on estimating and correctly reproducing static fixation points [4], [11], [9], [3]. In [18] a parameterized gaze controller has been proposed that allows several degrees of freedom for manual manipulation. Linear coupling between head and eye movement has been introduced in [10], while a rule based machine learning system for eye motion control has been proposed in [19].

Recently, attention has been given to the study of the brain control of the oculomotor system and attempts have been made towards the development of the corresponding neurobiological models. The reason behind the study of the brain control of the eyes are the several advantages that exist in the ocular system compared to other body systems that make it suitable for studying the neural control of coordinated, goal-directed movements. Eye movements can be measured accurately, since only six muscles control the position of each eye,and the circuits that control eye movements do not need to compensate for variable loads [16]. In particular, the saccadic system is the most intensely studied oculomotor subsystem.

Pioneer in oculomotor study is D.A.Robinson [13] [14] who put the basis on this scientific area. However, most of his work was limited in one dimensional movements. 3D eye movements have been studied thoroughly by Raphan [12]. In addition he has explored the important effects of extraocular muscle pulleys in determining saccade trajectory.

1.3 Motivation and contributions

The aforementioned approaches focus on the problem of fixation and smooth pursuit and almost neglect the significant saccadic nature of eye motions. This results in an incomplete model and non-realistic eye animations. The proposed framework makes a first step towards the virtual physiological modelling of the oculomuscular system for animation purposes. Based on seminal research of Robinson a computational model of the human eye and the associated muscles is provided that can be used to provide saccadic eye motions in a realistic manner. Therefore using as input target fixation positions realistic eye motions naturally emerge, without the need to manually edit the respective animation key-frames, as also demonstrated in the experimental results. The rest of the paper is organized as follows.

Section 2 presents an overview of the proposed framework. Section 3 describes details about the proposed computational oculo-muscular model, while Section 4 analyzes the muscle activation procedure. Section 5 describes the simulation procedure and results are presented. Finally, section 6 concludes the paper.

2 OVERVIEW

Figure 1 illustrates an overview of the proposed system. The backbone of the proposed system is the "simulation framework". It is based on a predefined oculo-muscular model that drives its dynamic behaviour and takes as input a formal definition of a specific task that can be as simple as a single fixation point or as complex as a dynamic trajectory. The action management module decomposes the input action in sequences of primitive tasks, each of which is handled separately. The kinematics manager and dynamics controller estimate the kinematic, i.e. position trajectories, and dynamic properties. i.e. torques that are needed to perform the action, respectively. Through an optimization procedure [1] the ocular torques are translated into oculu-muscular forces and muscle activation signals.

Then the resulted muscle forces are applied on the oculo-muscular model in a forward dynamics manner and the resulted motion trajectories are rendered. It should be emphasized that the realism of the proposed framework is highly dependent on the realistic oculomuscular model definition. For this purpose the model derived by the seminal work of Robinson is used and integrated in the dynamics equations of the simulation framework
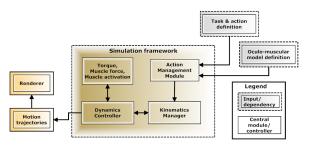

Figure 1: Overview of the proposed framework

Most of the aforementioned architectural blocks are described in the following sections. It should be mentioned that the implementation of the proposed framework is based on the OpenSim platform [1].

3 MODELING

Prior to the simulation of a physical system, the corresponding model, that has the features of the physical system we wish to study, has to be developed. It is nearly impossible to capture the full complexity of the physical system, so depending on the depth of detail we

demand and on the kind of simulation we are performing, several modelling choices have to be made. The goal of this paper is to produce realistic, physics-based animation of the human eyes, so the details we are interested in, have to do with the mechanics of the eye and in particular the way that the anatomy along with the dynamics of the six extraocular muscles affect the eye motion. Beyond the dynamics of the muscles, the passive tissues of the orbit have been proved to contribute greatly in saccades, mostly in the decelerating phase, rendering their inclusion to the final model necessary.

3.1 Anatomy

The human eye, with respect to the anatomical design of the globe can be safely considered for our purpose, a perfect symmetrical sphere that is able to rotate about its centre. The eye diameter for adults is 24 mm, with very little deviations between individuals, while the eye mass is 7.5 grams. Responsible for moving the eye are the six extraocular muscles.

Figure 2: Eyeball with extraocular muscles. Front/Side View

In the orbit we can find one more muscle that handles the elevation of the eyelid, but has no contribution to the globe movement. In Table 1 the characteristics of each one of the six muscles are shown. All muscles except Inferior Oblique have common origin point in the Annulus of Zinn, a ring of fibrous tissue behind the eye globe. The Annulus of Zinn is not right behind the globe, and this asymmetry will be included in our model, because it affects the moment arms of the applied forces. The four recti muscles have rather straightforward functionality in contrast to the two oblique muscles. With respect to the axis around which they can rotate the globe, the six muscles can be grouped in three pairs. The first pair is comprised by superior and inferior rectus and is able to rotate the eye upwards / downwards. The lateral and medial rectus form the second pair which can rotate the eye left / right. The two oblique muscles cannot be *assigned* a specific rotation axis, since their insertion points make them capable of contributing in virtually all eye movements. The exact coordinates of the origin or the insertion points of the muscles could not be reliably found in medical bibliography, so these parameters were picked by careful examination of images. Table 2 summarizes

all the geometrical points of interest. A graphical illustration of the model is depicted in figures 3, 4. In particular, in figure 4 the capability of the muscle to wrap round the eyeball is illustrated.
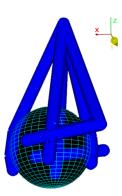

Figure 3: Illustration of the anatomy of the developed model of eyeball and extraocular muscles. Top View.
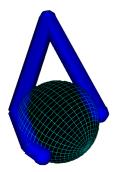

Figure 4: Lateral Rectus wrapping round the eyeball.

3.2 Dynamics

In order to create a complete model of the eye, that will be capable to provide the necessary realism regarding the trajectory of the movement, in response to some desired eye orientation, the dynamics of the system components should be obviously taken into consideration. The components that affect the movement are the muscles and the passive tissues that surround the eye globe. The dynamic mechanical response of the body muscles is an active research field. Most of the existing models originate from the [5] Hill Muscle Model, a three-element model that manages to reflect the viscoelastic nature of the muscles. However, up to now, focus has mainly been given to the study of larger body muscles, responsible for actuating the skeleton and therefore the tuning of the model parameters has been based on those kinds of muscles. Unfortunately, these models appear not be suitable for the extraocular muscles and a reason for that, is the way in which the eye should be able to move, that is precise and swift when it performs saccades, or slow in smooth pursuit situations. On the other hand, that high precision is not usually required

Muscle	Origin	Adduction	Abduction
Superior Rectus	Annulus of Zinn	Intorsion, Adduction, Elevation	Elevation
Inferior Rectus	Annulus of Zinn	Extorsion, Adduction, Depression	Depression
Lateral Rectus	Annulus of Zinn	Abduction	-
Medial Rectus	Annulus of Zinn	Adduction	-
Superior Oblique	Annulus of Zinn*	Intorsion	Intorsion, Abduction, Depression
Inferior Oblique	Maxillary Bone	Extorsion	Extorsion, Abduction, Elevation

Table 1: Motion Capability of Extraocular Muscles. *Superior Oblique's path goes through the trochlea*

	X	Y	Z
Annulus of Zinn	-6	0	36
Maxillary	-14.4	-9.6	0
Troclea	-14.4	12	2.4
Medial-Rectus	-10.8	0	-5.4
Lateral-Rectus	-10.8	0	-5.4
Inferior Rectus	0	-10.8	-5.4
Superior Rectus	0	10.8	-5.4
Inferior Oblique	5.4	10.8	-5.4
Superior Oblique	5.4	-10.8	-5.4

Table 2: Coordinates of origins and insertion points of Muscles and Pulleys (mm). Coordinates in this table refer to the right eye. Left eye's coordinates are the same but mirrored in the YZ plane ($X_{left} = -X_{right}$). The origin is on the eyeball's center, and the axes are shown in figure 3.

by most body muscles. A set of differential equations that describe the behaviour of the extraocular muscles *during one dimension saccades* have been proposed in [13]. Equation 1 is Newton's 2nd law, while equations 2 and 3 describe the muscle activation dynamics and the passive behaviour of the system respectively. By *muscle activation dynamics* we refer to the delay that is observed between the excitation signal carried by the neurons to the muscles and the actual force development by means of muscle contraction.

$$m\frac{d^2\Theta}{dt^2} = F_p + F_m \qquad (1)$$

$$\frac{R_m}{K_e}\frac{dF_m}{dt} + F_m = F_0 - R_m\frac{d\theta}{dt} \qquad (2)$$

$$R_1 R_2 \frac{d^2\Theta}{dt^2} + (R_1 K_2 + R_2 K_1)\frac{d\theta}{dt} +$$
$$K_1 K_2 \theta = (K_1 K_2)F_p + (R_1 R_2)\frac{dF_p}{dt} \qquad (3)$$

θ:	Eye angle
F_m:	Net Added Muscle force
F_p:	Net Passive force
F_0:	Net active state tension
R_m:	Net muscle force velocity slope
K_e:	Net muscle elastic stiffness.
K_1, K_2:	Stiffness of the passive springs
R_1, R_2:	Viscosity of the passive springs

The passive force part contains *all* the passive elements of the plant. Some belong to the muscles, while others belong to the orbit. Muscle passive elements are the sarcolemma, sarcoplasmic reticulum, perimysium, endomysium, vasulature. Orbital passive elements are the Tenon's capsule, orbital fat, conjuctival tissue, check and suspensory ligaments. The reason for the two constants of stiffness and viscosity is that Robinson's proposed model [13] has two viscoelastic elements connected in series, the first having a slow time constant and the second a fast one. The distinction that is made between passive and active part of the model causes some obscurity concerning which forces are generated by the muscles and which by the passive tissues. Robinson's approach, which we follow as well, is to lump the passive part of the muscles together with the rest passive elements, leaving the muscle to contribute only the, so called, **added force**. By added force, Robinson refers to the portion of the force that the muscle would not normally apply if nervous excitation did not exist and derives from the chemical processes that cause the muscle to contract. On the other hand, the passive force is always there deriving from the material characteristics of the muscle and would apply even if the muscle was dead.

The concept of isolating the added force makes it possible to model the muscles as ideal actuators with the force that they develop being linearly proportional to the control input they receive. The remaining passive part of the model was modelled with three spring-damper elements (Stiffness, Viscosity), one for every degree of freedom of the eyeball. On Z axis, both stiffness and viscosity were multiplied by a coefficient relatively in order to reduce the transient torsional rotations that were observed to agree with Listing's Law [17]. In figure 5 this is shown. The black line is the torsional
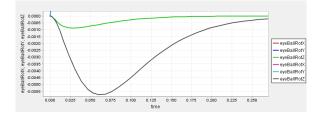
Figure 5: Non-zero transient torsional component of rotation. With the introduction of the increased stiffness on Z axis' elastic force the torsions were notably decreased.

rotation on a saccade without the increased coefficient, while the green one is the same quantity after the increment of Z axis' stiffness.

There are two reasons why we did not model directly the equations 1-3. Firstly, the numerical integration of a more complex system would be less efficient and would make more difficult the use of the proposed system on applications that the saccade trajectory would be required in real-time. Such applications include video games and VR simulators. Secondly, the problem of scaling the one-dimension dynamic equations to the thee dimensions is nothing but trivial.

4 MUSCLE ACTIVATION

In section 3 we developed an anatomical/dynamic model of the human eye. Still, the most important part in order to make a saccade simulation is missing. As we already mentioned, the active force developed by the muscles is controlled by the nervous system, with the later determining the force requirements for a given saccade, and subsequently supplying the appropriate muscle excitations signals. That said, it becomes clear that an adequate model of just the dynamic part of the system is not enough to produce a realistic saccade. We have to obtain knowledge about the way the Nervous System drives the muscles in order to somehow imitate it. Hopefully, this process will result to a realistic animation that looks convincing to a human viewer. Again, we use the results of Robinson's work, where he makes some conclusions about the forces before, during and after a saccade.

4.1 Steady state muscle activation

At first is should be stated that the 6 extraocular muscles are never in rest. At every fixation point, they should equilibrate the elastic restoring forces of the passive tissues and additionally keep the eye steady and stiff at its position. In Robinson's one-dimensional muscle-abstract approach, the identification of the equilibrating forces is trivial. In our approach this is not the case. Especially when 3D eye orientations are considered and due to the redundancy of actuators with respect to the

DoFs, there are multiple valid solutions regarding the muscle utilization that satisfy the constraint of the specified eye orientation. To overcome this issue, we make use of an optimization algorithm. This algorithm tries to find the solution that minimizes the sum of squares of muscle activation levels under, of course, the orientation constraint:

$$ssa = arg\{\sum(ssa_m^2) \mid Fixed\ Orientation\}, \quad ssa \in R^6$$

where ssa is a vector containing the steady-state muscle activation of all muscles. This way, the quantity that is actually minimized is the total energy of the system, a fact that has been found to be valid in many movement scenarios.

4.2 Transient muscle activation

At this point we have successfully determined the steady-state activations before and after the sacacade given the two orientations (initial/final). The question is, how do the muscle activation levels change from the initial state to the final, so that the eye will rapidly and precisely make the transition to the new orientation. Robinson's findings suggest that this does not happen via the application of a step function that would instantaneously change the neural signal to the new steady state level, because if that was the case, due to the highly overdamped mechanical response of the eyeball system, the saccade would complete in a very long time. If that was the case, the saccade trajectories would be like in figures 6, 7. According to [13] there is no active control during the movement. What this means, is that the eyeball system is an open-loop system and that muscle activations as functions of time are already *determined* by the brain before the movement begins.

Figure 6: Step response. 10°horizontally. Total saccade duration is nearly 200ms, which is far above the observed durations in human subjects (<100ms).

4.3 Activation signal details

Trying to drive our simulation by to the human physiology, we constructed a muscle activation signal that succeeds to reproduce most of the features of Robinson's experimental results of saccades on real humans. The total duration of the saccade is divided into three phases.
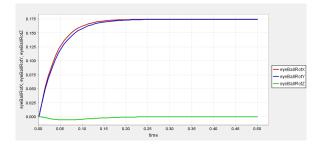
Figure 7: Step response. 10°horizontally & vertically. Total saccade duration is more than 160ms, which is far above the observed durations in human subjects (<100ms).

1. *Excess force phase:* For a brief time period in the beginning of the movement, an activation signal larger than the steady state level is applied to every muscle resulting to an excess force greater than required to keep the equilibrium. According to Robinson, this also happens in real human saccades, and aims to accelerate the saccade. The magnitude of the excess activation is proportional to the difference between initial and final activation levels, while the duration depends on the 3D angle between initial and final orientation. These are the parameters of the excess activation:

$$dssa = ssa_{after} - ssa_{before}$$
$$xc_{dur} = 25ms + angle \times 0.2ms/°$$
$$xc_{magn} = dssa \times 0.75$$

where xc_{dur} is the duration of application of the excess activation, xc_{magn} is the magnitude of the excess activation level and ssa_{before}, ssa_{after} are the initial and final steady state activation levels respectively.

2. *Error phase:* After the excess force period, the activation signal drops at a_{error} level which is the steady-state level minus a small error. That way, we capture the error of the initial calculation that is corrected with a small "fix" saccade after the completion of the main one. This phenomenon has been observed on humans, especially on large saccades.

$$a_{error} = ssa_{before} + dssa * 0.975$$

3. *Fix phase:* The last phase is the "fix" phase. Activation settles to the steady state level after applying a spike of activation to every muscle in order to quickly "fix" the error of the previous phase.

Every human does not have the same response times regarding saccades. Moreover, in a specific individual different behaviour can be observed depending on many factors like mood, fatigue etc. With appropriate configuration of the signal parameters, one can modify the produced saccades in order to simulate the variations in saccade response. The parameters we have chosen are the following:
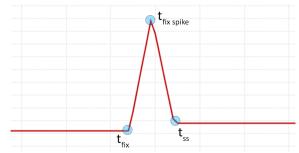

Figure 8: Fix signal details.

$$t_{fix} = 100ms + angle \times 0.5ms/°$$
$$t_{fixspike} = t_{fix} + 5ms$$
$$t_{ss} = t_{fix} + 10ms$$

In figure 8 the above parameters are illustrated.

5 SIMULATION RESULTS

For the dynamic computations our framework is based on the OpenSim platform [1], [15].

5.1 Simulation procedure

The simulation procedure is the following: We take as inputs the initial and target orientation of the eye. For both orientations we calculate the rotations in X,Y axes using Euler Angles representation. The torsional rotation (Z) in steady state is set to zero as Listing's Law dictates. The next step is to invoke the static optimization algorithm in order to compute the steady state activation levels and feed them to the routine that constructs the activation signals. The last step is to perform a forward dynamic simulation by initializing the orientation of the model appropriately and applying the computed activation signals to each muscle. The resulting eye trajectory is used to animate a 3D eye model. Plots of the resulting trajectories for different initial and final orientations follow.

5.2 Saccade plots

Figures 9 - 10 are plots of eye rotation angles about the three axes. *eyeBallRotX* represents the vertical rotation, *eyeBallRotY* represent the horizontal rotation and *eyeBallRotZ* the rotation about the line of sight. All rotations are in degrees. Abscissa represents time in seconds. Figure 9 illustrates a horizontal right-to-left saccade of relatively large total rotation angle (40°). Due to the large angle, a small overshoot is observed, that agrees with existing oculomotor studies [13]. Figure 10 shows an oblique saccade of 20 °horizontal component and 5 °vertical. Overshoot is not observed on the horizontal rotation, while there is a small overshoot to the vertical rotation that is caused by the non-zero torsional rotation. Regarding the fix saccade, the horizontal component has a larger magnitude because the total angle of
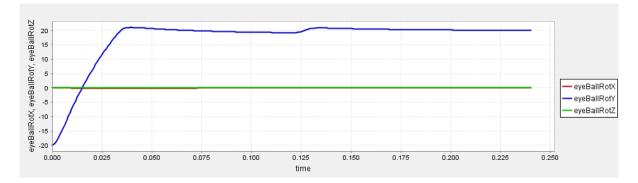
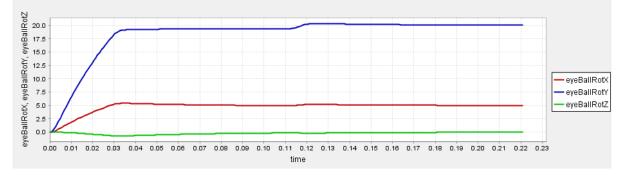Figure 9: Horizontal saccade simulation. $(0°, -20°) \Rightarrow (0°, 20°)$

Figure 10: Oblique saccade simulation. $(0°, 0°) \Rightarrow (5°, 20°)$

Figure 11: Saccade: $(0°, 0°) \Rightarrow (0°, 10°)$

Figure 13: Saccade: $(0°, 30°) \Rightarrow (0°, 10°)$
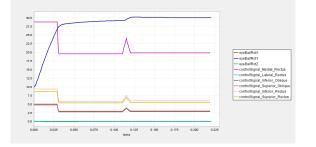

Figure 12: Saccade: $(0°, 10°) \Rightarrow (0°, 30°)$

Figure 14: Saccade: $(10°, 10°) \Rightarrow (-10°, -10°)$

rotation on that direction was 4 times greater than the corresponding angle on vertical direction.

In Figures 11 - 14 several saccade trajectories are illustrated for various initial and final orientations. Again, rotations about the three axes are plotted versus ti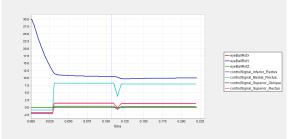me. On these figures, the control signals that were used to drive the simulations have been overlaid on top of trajectories. The scale of the control signals is qualitative. Control signals are shown only for the muscles that take part at each saccade i.e. have positive activation level. In some cases the activation calculation module outputs small negative activation values. These are

clamped to zero by our muscle model, as a *muscle actuator* can only support the pulling action (positive activations) and can never push (negative activation). The three phases of the saccade can be easily spotted. In the fist phase we have the excess activation that accelerates the sacacde, then the error settling phase, and finally the fix activation spike followed by the steady-state activation levels. The trajectory plots reveal that, the goal of simulating a realistic trajectory has been accomplished. Eye physiology results from [13], [14] and [12] agree with our results in trajectory shape, rise times, magnitude of fix microsaccade, absence of important torsional rotation and duration.

6 CONCLUSION

The major contribution of this paper is an attempt to build a realistic biomechanical human eye model following the principles of the virtual physiological human. Anatomical details of the oculo-motor system were incorporated both in the modeling and in the dynamics simulation of the eye motor behavior. As a result, based on the corresponding muscle activation signals, realistic eye saccadic motions naturally emerge. Experimental results demonstrate that the proposed framework provides motion trajectories that are very close to the seminal clinical studies of Robinson. Even if the accuracy and fidelity of the proposed approach is proven remarkable, it does not deal with the problem of incorporating volumetric tissue deformations that still remains an open problem. However, the simplified model of 1D muscular elements that is seen to be very realistic in the past in musculoskeletal simulations, proves itself also in the case of oculo-motor simulations, with the addition however, of a relatively complex underlying physiological model.

7 ACKNOWLEDGEMENTSS

8 REFERENCES

[1] S. L. Delp, F. C. Anderson, A. S. Arnold, P. Loan, A. Habib, C. T. John, E. Guendelman, and D. G. Thelen. OpenSim: open-source software to create and analyze dynamic simulations of movement. *IEEE transactions on bio-medical engineering*, 54(11):1940–1950, 2007.

[2] Z. Deng, J. P. Lewis, and U. Neumann. Automated Eye. (April), 2005.

[3] Z. Deng, J. P. Lewis, and U. Neumann. Automated eye motion using texture synthesis. *IEEE Computer Graphics and Applications*, 25(2):24–30, 2005.

[4] E. Gu and N. I. Badler. Visual attention and eye gaze during multiparty conversations with distractions. In *International Conference on Intelligent Virtual Agents 2006*, pages 193–204, 2006.

[5] a. V. Hill. The Heat of Shortening and the Dynamic Constants of Muscle. *Proceedings of the Royal Society B: Biological Sciences*, 126(843):136–195, Oct. 1938.

[6] L. Itti, N. Dhavale, and F. Pighin. Realistic avatar eye and head animation using a neurobiological model of visual attention. *Optical Science*, 2004.

[7] B. H. Le, X. Ma, Z. Deng, and S. Member. Head-and-Eye Motion Generators. X(X):1–14, 2012.

[8] S. P. Lee, J. B. Badler, and N. I. Badler. Eyes alive. *ACM Transactions on Graphics*, 21(3):637–644, July 2002.

[9] S. P. Lee, J. B. Badler, and N. I. Badler. Eyes alive. In *SIGGRAPH*, pages 637–644, 2002.

[10] X. Ma and Z. Deng. Natural eye motion synthesis by modeling gaze-head coupling. In *IEEE VR 2009*, pages 143–150, 2009.

[11] S. Masuko and J. Hoshino. Head-eye animation corresponding to a conversation for cg characters. *Computer Graphics Forum*, 26(3):303–312, 2007.

[12] T. Raphan. Modeling control of eye orientation in three dimensions. I. Role of muscle pulleys in determining saccadic trajectory. *Journal of neurophysiology*, 79(5):2653–2667, 1998.

[13] B. Y. D. A. Robinson. The mechanics of human saccadic eye movement. pages 245–264, 1964.

[14] D. A. Robinson, D. O. Meara, A. B. Scott, and C. C. Collins. components of human eye movements. 26(5), 1969.

[15] M. a. Sherman, A. Seth, and S. L. Delp. Simbody: multibody dynamics for biomedical research. *Procedia IUTAM*, 2:241–261, Jan. 2011.

[16] D. L. Sparks. The brainstem control of saccadic eye movements. *Nature reviews. Neuroscience*, 3(12):952–64, Dec. 2002.

[17] D. Straumann, D. S. Zee, D. Solomon, and P. D. Kramer. Validity of Listing's law during fixations, saccades, smooth pursuit eye movements, and blinks. *Experimental brain research*, 112(1):135–46, Nov. 1996.

[18] R. time expressive gaze animation for virtual humans. Eyes alive. In *AAMAS 2009*, pages 321–328, 2009.

[19] R. Vertegaal, R. Slagter, G. van der Veer, and A. Nijholt. Eye gaze patterns in conversations: there is more to conversational agents than meets the eyes. In *CHI 2001*, pages 301–308, 2001.